Section 1.2 Experiment, Sample Space and Random Event

1.2.1 Basic Definitions

A random phenomenon is a situation in which we know what outcomes could happen, but we do not know which particular outcome did or will happen. ‘Random’ in statistics is not a synonym for ‘haphazard’ but a description of a kind of order that emerges only in the long run.

Experiment, Sample Space and Random Event

- 1.2.1 Basic Definitions
- 1.2.2 Events as Sets

1.2.1 Basic Definitions

Random experiment usually has the following three characteristics.

(i) Repeatability: it can be repeated under the same conditions.

(ii) Predictability: it has more than one outcome and we know all possible outcomes before the experiment.

(iii) Uncertainty: the outcome of the experiment will not be known in advance.

In this book, we shall abbreviate “random experiment” to experiment and denote it by E.

Some examples of random experiment:

E_1 : Determination of the sex of a newborn child.

E_2 : Roll a die and observe which number appears.

E_3 : Flip two coins and observe the outcomes.

E_4 : Roll two dice and observe the outcomes.

E_5 : Observe call times for a call center.

E_6 : Measure the lifetime of cars.

How to record the experiment data?

Each possible outcome: ***sample point*** (ω)
The set of all possible outcomes: ***sample space*** (Ω)

1.2.1 Basic Definitions

Example

E_1 : *Determination of the sex of a newborn child.*

For E_1, $\Omega_1 = \{g, b\}$, where the outcome g means that the child is a girl and b that it is a boy.

Example

E_2 : *Roll a die and observe which number appears.*

For E_2, $\Omega_2 = \{1, 2, 3, 4, 5, 6\}$, where the outcome i means that i appeared on the die, $i = 1, 2, 3, 4, 5, 6$.

Example

E_3 : *Flip two coins and observe the outcomes.*

For E_3, $\Omega_3 = \{(H, H), (H, T), (T, H), (T, T)\}$, where H means head and T means tail.

Example

E_4 : *Roll two dice and observe the outcomes.*

For E_4,

$$\Omega_4 = \begin{pmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{pmatrix}$$

where the outcome (i, j) is said to occur if i appears on the first die and j appears on the second die, $i, j = 1, 2, \cdots, 6$.

1.2.1 Basic Definitions

Example

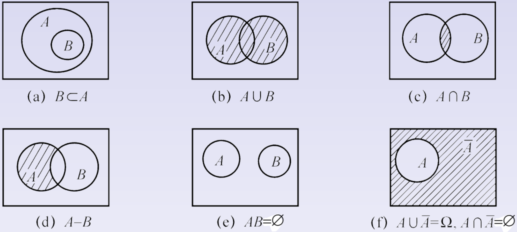
E_6 : *Measure the lifetime of cars.*

For E_6, $\Omega_6 = [0, \infty)$, where the outcome t is the lifetime of a car, $0 \leqslant t < \infty$.

- Any subset of the sample space Ω is known as an ***random event*** or ***event***. Events are usually denoted by capital letters $A, B, C, \cdots$.
- Those events must occur in the experiment are called the ***inevitable events***.
  Sample space Ω is an inevitable event.
- Those could not happen anytime are said to be ***impossible events***. We usually denote impossible events by $\emptyset$.

1.2.2 Events as Sets

Figure: The relationships between events A and B



$$\bigcup_{i=1}^{n} A_i \triangleq A_1 \cup A_2 \cup \cdots \cup A_n, \quad \bigcup_{i=1}^{\infty} A_i \triangleq A_1 \cup A_2 \cup \cdots$$

$$\bigcap_{i=1}^{n} A_i \triangleq A_1 \cap A_2 \cap \cdots \cap A_n, \quad \bigcap_{i=1}^{\infty} A_i \triangleq A_1 \cap A_2 \cap \cdots$$

1.2.2 Events as Sets

Table: The jargons in set theory and probability theory

notation	Set jargon	Probability jargon
Ω	Collection of objects	Sample space
ω	Member of Ω	Elementary event, outcome
A	Subset of Ω	Events that some outcome in A oc
A or $\overline{A}$	Complement of A	Event that no outcome in A occurs
$A \cap B$	Intersection	Both A and B
$A \cup B$	Union	Either A or B or both
$A - B$	Difference	A, but no B
$A \subseteq B$	Inclusion	If A, then B
$\emptyset$	Empty set	Impossible event
Ω	Whole space	Certain set

1.2.2 Events as Sets

Let A, B and C be the random events of experiment E. The operations of the events will satisfy the following rules:

(i) Commutatively $A \cup B = B \cup A$, $AB = BA$.

(ii) Associatively $A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$.

$A(BC) = (AB)C = ABC$.

(iii) Distributively $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,

$A(B \cup C) = (AB) \cup (AC)$.

(iv) De Morgan's law $\overline{A \cup B} = \overline{A} \cap \overline{B}$, $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

More generally, $\overline{(\bigcup_i A_i)} = \bigcap \overline{A_i}$, $\overline{(\bigcap_i A_i)} = \bigcup \overline{A_i}$.

1.2.2 Events as Sets

In addition, there are some common properties, such as

(1) $\overline{\overline{A}} = A$.

(2) $A \cup B \supset A$ and $A \cup B \supset B$. In particular, if $A \subset B$, then $A \cup B = B$.

(3) $A \cap B \subset A$ and $A \cap B \subset B$. In particular, if $A \subset B$, then $A \cap B = A$.

(4) $A - B = A - AB = A\overline{B}$.

(5) $A \cup B = A \cup \overline{A}B$.

1.2.2 Events as Sets

Example

Suppose that A, B and C are three events and $D = \{$at least one of the three events will occur$\}$. Try to describe the event D by events A, B and C.

Solution. We describe the event D by three different ways:

(i) directed method: $D = A \cup B \cup C$,

(ii) decomposition method:

$D = A\overline{B}\ \overline{C} \cup \overline{A}B\overline{C} \cup \overline{A}\ \overline{B}C \cup \overline{A}BC \cup A\overline{B}C \cup AB\overline{C} \cup ABC$,

(iii) inverse method:

$D = \overline{\overline{D}} = \overline{\{A,\ B \text{ and } C \text{ will not occur}\}} = \overline{\overline{A}\ \overline{B}\ \overline{C}}$.

1.2.2 Events as Sets

Example

Select an integer from 1 to 1000 at random. How to describe the event that the integer is not divisible by 6 and 8.

Thank you for your patience !